Kuwait University | Math 102 | March 27, 1997
Dept. of Math. and Comp. Sc. | First Examination | Duration: 75 minutes

Calculators and mobile phones are not allowed.

Answer all of the following questions.

Each (sub)question is worth 4 points.

1. (a) Let

$$f(x) = \tan^{-1}(1 + 5x^2) \qquad (x \geq 0)$$

Prove that f is one-to-one and find its inverse. Also state the domain and range of the inverse function.

(b) Let

$$g(x) = \int_2^x \frac{t\,dt}{e^{t-2} + t^4} \qquad (x \geq 0)$$

Show that the g is one-to-one and find the equation of the tangent line to the graph of its inverse function g^{-1} at the point $P(0, 2)$.

2. Find $\dfrac{dy}{dx}$ if

(a) $\ln|x + e^{xy+2y}| - xy = 2x^2 - y^2$

(b) $y = (\cos x)^{\cos x} + \cos^{-1}(\log_3 x)$

(c) $y = \sqrt{\dfrac{\sec x \tan x}{\sqrt{\ln x + 2^x}}}$.

3. Show that the function defined by

$$f(x) = \arcsin x - 2\arctan\sqrt{\frac{1+x}{1-x}} \qquad (0 \leq x < 1)$$

is a constant function.

4. Write $\sec\left(\sin^{-1}\left(\dfrac{1}{x}\right)\right)$ as an algebraic expression in x if $x > 1$.

5. Evaluate the following integrals

(a) $\displaystyle\int \frac{dx}{(x^2+1)\tan^{-1} x}$ (b) $\displaystyle\int \frac{1}{|1 + e^{-2x}|}dx$, (c) $\displaystyle\int (\tan(1-2x) + x\sec x^2)dx$,